Abstract

The design and implementation of a projection system that has the ability to generate a three-dimensional model from the screen of a mobile device is presented. On devices today is the issue of a limitation as to the amount of content that can be displayed on screen; as this depends on the proportions of each device. There are methods for three-dimensional reproductions, here a simple and economical way to get it is proposed. It is has developed a prototype which is adaptable to a mobile device. An algorithm to generate four different views of a figure is planned. For which tests were performed with VRMLandRedMan. Note that most of the holographic projectors have four identical views of an object, but this can not consider it like a hologram. In our case the coordination of movements in the four different faces is remarkable.

1. Introduction

Today you have several ways to recreate holograms, getting a three-dimensional effect on the vision of the people. A picture is only the reflection of light by pigmentation containing any surface where only certain frequencies of light waves are visible to the eye, which are perceived by the vision and represented by a color. Holography uses this perception to recreate an object that does not exist [1].

Holographic projectors used are not adapted for use on a mobile device. Based on projectors that are on the market and some research, a method for generating 3D images is developed [2].

The amount of future and final applications are large, from an analysis of a three dimensional object to the creation of virtual worlds for simulations; Thanks to the portability of the device mobile is achieved show anytime and in many places.

Using projection devices is necessary today to display detailed information; ergo show three-dimensional models, and interact with them, for better understanding of the object projected; whether for education, engineering, medicine, and even marketing [3] [4]

3. Holography and dimensionality

The term holography was invented in 1947 by Dennis Gabor [5], comes from the Greek word "holos" which means "engraving full" is a technique of 3D sampling involves using the interference and diffraction on a plate engraved and reconstruction of optical wave front.

There are different ways to create holograms for example etching interferometer, which has evolved from a photograph which includes features of depth of the object recorded in a polymer, to perform reconstruction of the image with light instead of using means the reference light, or information recorded reference to its reconstruction.

This trend has been changing depending on the type of holography technique required; in the optical field has made several advances, getting pretending fool the human eye perceive depth. One of these developments is the creation of stereograms.

In Figure 1 are classified, different types of three-dimensional displays, ranging from simple optical techniques to the laser volumetric models [6], does not take into account the model liquid crystal display technology (LCD) and similar, since it is necessary to use special instruments [6].

4. Mathematical justification of the optical effect

If we have a beam of light that goes from point A to point B, and this is reflected on a surface, the trajectory according to the principle of Fermat [3] should be the minimum. Taking a medium that does not suffer from disturbances, then the speed of light is constant in the medium. In order to find the minimum beam path we need to find the optical route [3] and this can be represented as in Figure 2.
As seen in Figure 27, a, b and c are constant distances, the angle of incidence \( i \) is also constant, and the angle of reflection \( r \) depend on the result of our analysis, the speed of propagation of the light beam is constant and that not happen in different ways, but the medium is kept constant and therefore also the propagation velocity; Assuming that the time is directly proportional to the distance and inversely proportional to the speed of the object (in this case light beam) we have that total propagation time of light beam from point A to point B is:

\[
 t = \frac{d}{v} \tag{1}
\]

Where:
- \( t \) = time
- \( d \) = distance
- \( v \) = velocity

Then we deduce that, from (1), the maximum time it takes to traverse the beam is calculated for Fig. 2:

\[
 t = \frac{AC}{v} + \frac{CB}{v} \tag{2}
\]

Where:
- \( (AC) \) = Distance from point A to point C
- \( (CB) \) = Distance from point C to point B
- \( v \) = The speed in the middle

From Figure 27 we can define the value of \( (AC) \) and \( (CB) \), we substitute in (2) having the following equation:

\[
 t = \frac{1}{v} \left[ \frac{c-x}{\sqrt{(c-x)^2 + a^2}} + \frac{x}{\sqrt{x^2 + b^2}} \right] \tag{3}
\]

To find the minimum optical path that satisfies the requirements, the time derivative is calculated with respect to the position of the light beam in the equation (3):

\[
 \frac{dt}{dx} = \frac{1}{v} \left[ -\frac{c-x}{\sqrt{(c-x)^2 + a^2}} \right] \frac{1}{\sqrt{x^2 + b^2}} \tag{4}
\]

Of Fermat principle we have:

\[
 \frac{dt}{dx} = 0 \tag{5}
\]

Solving and equating equations (4) and (5) yields equation (6):

\[
 \frac{dt}{dx} = \frac{1}{v} \left[ -\frac{c-x}{\sqrt{(c-x)^2 + a^2}} + \frac{x}{\sqrt{x^2 + b^2}} \right] \tag{6}
\]

Using the equivalence of sin on equation (6) we obtain:

\[
 \sin(i) = \sin(r) \tag{7}
\]

From equation (7) we can deduce that the angle of incidence \( i \) is equivalent to or reflective angle \( r \):\n
\[
 i = r \tag{8}
\]

We can define an ideal angle for the projection of an image of a device onto a surface for the recreation of images in space.
Software is designed to display information optimally for the appreciation of the viewer and that do not lose the perception of the virtual object.

5.1. Project design

Given the mathematical models is proposed an optical design. Ideally, a model of N faces (figure 4). Where N tends to infinity, but economically and experimentally is difficult to create this model. The Model was employed four faces; as the projection level no greater complexity by increasing the faces (in terms of design and hardware). We are proposing for a 45-degree angle to the initial reflection, which is justified by the results.

Figure 4. Proposal pyramid N (N = 20) sides.

The pyramid structure can support the weight of the device (figure 5a), a rotation of 180 ° about the shaft by “y”. Pyramid is designed for that the mobile device can support the weight of it (figure. 5b).

a)  

Figure 5. (a) Mobile device at the top. (b) Pyramid on your mobile device.

Instead of occupying specific projectors, are projected images by each side using the display device (figure 6). This ensures that the image projected by the mobile device is the image in the pyramid.

5.1. Mathematical foundation of model creation

For the calculation of the model of Pyramid projection, we will base on the angle α (45 degrees); this data has the information needed to define an optimal model. Since we have a main angle of incidence of the image, a model like the one in Figure 7 is proposed.

Figure 7. Proposed mathematical model pyramid.

If we have that θ is:

\[ \theta = \tan^{-1} \left( \frac{\left( \frac{a^2}{4} + b^2 \right)^{\frac{1}{2}}}{\frac{a}{2}} \right) \]  

(9)

Solving equation (9) we obtain:

\[ \theta = \tan^{-1} \left( \frac{\left( \frac{a^2}{4} + b^2 \right)^{\frac{1}{2}}}{\frac{a}{2}} \right) \]  

(10)

If α=45° based on the projection angle, then it follows from the figure that

\[ \tan(\alpha) = \frac{b}{\frac{a}{2}} \]  

(11)

Solving equation (11) we have that b is
Therefore, solving for \( b \) of the equation (10) gives that

\[
\theta = \tan^{-1}(\sqrt{2})
\]

Then \( \theta \) is equivalent or have a result:

\[
\theta = 54.73^\circ
\]

Besides thanks to obtain equation (12) that the height \( b \) of the face of the pyramid is

\[
\left(b^2 + \frac{a^2}{4}\right)^{1/2} = \left(2 \cdot \frac{a^2}{4}\right)^{1/2}
\]

Solving we obtain that the height has a value of:

\[
\left(b^2 + \frac{a^2}{4}\right)^{1/2} = \frac{a}{\sqrt{2}}
\]

In conclusion, each face of the pyramid has a height of

\[
h = \frac{a}{\sqrt{2}}
\]

And base “a”

For maximum utilization of materials calculations are kept solving the variable height from a square of dimensions \( 2 \times h \times 2 \times h \), taking the arrangement in figure 8 for cuts:

![Figure 8. Design cuts the material.](image)

The gray area is the material that does not address the creation of the pyramid (fig. 33). This yields an optimal use of the material for a design of a pyramid; in addition a 10% cut of the tip of the pyramid for his support on the mobile device. See Figure 5b. You lose only 14% of capacity projection display, and experimentally have added support.

### 5.1. Projection design software

For software design two types of projection used languages: Render Man Interface byte stream (RIB) [7] [8], which is the most common format for data entry scene to render compiler; and one mobile processing support for modelling in virtual 3D worlds (VRML). In the RIB projection tests were performed, as seen in Figure 9, and basic models that could play without a lot of code were projected.

![Figure 9. Two spheres on a plane on the z axis.](image)

Tests were conducted with a second code, resulting in a projection algorithm, which shows all the characteristics of the object regardless of the type; even if you suffer a change in rotation to continue receiving this answer that the object is inside the pyramid.

This cube of Figure 10 resolves uniqueness all its faces are different colors; front so we can see three of the colors that make up the cube in the back of the object we see the three missing features.

![Figure 10. Six-sided of cube in different colors.](image)

Projection regions (Figure 11) are then defined.

![Figure 11. Regions projection.](image)
However, of the views on the real world have different rotation in the virtual world; for this reason you must adjust the algorithm to adapt to the virtual world; ergo, having a mirrored model every point coincides with the law of reflection as we can see in Figure 12.

![Figure 12. Reflection objects.](image)

The principle of reflection states that the incident angle of a projected point is equal to the reflected angle of that point; ergo if our Figure A - D has a certain order in the virtual world in the real world it changes. As can be seen the ends B - C and D - E are reversed in the z axis, so that individual rotations are used for each of the faces of the virtual object (Figure 13).

![Figure 13. Processing of the front view.](image)

Once made all the adjustment of each of the faces; you have to arrange the details of the rotation; VRML language works with javascript, which allows us to change parameters of the algorithm for execution and real-time rendering, VRML has a rotation sensor; depending on how the object in the virtual world (Figure 14), each of the views should turn to react to this information and make correct change; rotation data are entered through the touch screen sensor and mobile device; it is captured in variables assigned depending on the direction of rotation.

![Figure 14. Direction of rotations in VRML.](image)

Each of the views on the real world have different rotation in the virtual world; for this reason you must adjust the algorithm to adapt to the virtual world; ergo, having a mirrored model every point coincides with the law of reflection as we can see in Figure 12.

![Figure 15. Algorithm Part One.](image)

First a universe defined; that is a virtual space in which to insert the objects; after the prototype view is defined, these in turn each contain an embedded prime universe to universe; that is, each of these virtual objects suffers rotation to achieve the effect of perception of the virtual object in the real world. In the next step of the algorithm (figure 16), the code sets the initial view of the main universe, this can vary from virtual objects inserted; once done the projection positions in each of these is defined and sent to call the functions corresponding to each prototype; that is, the front position is commanded to call the function declaration front view, left with the function declaration left view and so on. Each...
of these is moved a certain number of units in the main universe on the value of the x-axis or y-axis depending on the view. Wherein a rotation sensor for each of the views, which extract rotation information via touch screen sensor of the mobile device.

Figure 16. Algorithm Part Two.

The algorithm of rotation takes in each of the views, the rotation information concerning the view to which it belongs; ergo sensor front view extracts data from the rotation of the front view (Figure 17). The data obtained from the sensor corresponding to the amount of rotation of the object in x, y, z, and the degrees of rotation φ. These values will be stored in variables, which will help us to change rotations in other views.

To the corresponding view, the values of the variables do not change; for the left eye with respect to the current view, the value of x, making the inverted value of the variable and the value of y, making the inverted value of the variable z, the value of z, takes the value of invested variable x, and main- tains the variable φ; for the right view, the value of x, takes the value of the variable, the value of y, making the inverted value of the variable z, the value of z, making the inverted value of the variable x, and the variable φ it is preserved; In the rear view, the value of x, takes the value of the variable x, the value of y, making the inverted value of the variable and the value of z, making the inverted value of the variable z, and φ varying remains.

6. Results
6.1. Screening tests
To confirm that the proposed panel is capable of projecting a virtual object, the three-dimensional model of a cone was used (figure 18). Four identical virtual objects were placed, and the algorithm in Figure 40 is applied, adding the rotation of 0, π, π / 2, and -π / 2 axis “z” in the respective model. It achieves optimal projection shows that the optical image sent process is performed successfully.

Figure 18. Cone made in RIB.

If the model has more views than the pyramid, not always all viewpoints are satisfied; and besides, we can see that after a viewing angle we do not perceive the full projection; this affects the display characteristics information of the object (figure 19).
Another test was performed to fill the interior walls to perform the projection (figure 20); this does not yield a satisfactory result. Although the second half to be contaminated, if very small, the effect of internal refraction is irrelevant, since impurities are minimal or insignificant, this causes the deviation is minimal.

This behavior can be justified mathematically. It has to $\beta$ is the angle of incidence of the signal (figure 21) with respect to the normal (N) of the reflecting surface; the information sent, based on the theory of reflection and the law of Snell, and is refracted and reflected, if we take each pixel of the object as a point source of light, the first point in passing through the second, air and glass, respectively, suffering from a deflected, both speed and direction. The material is not uniform; the common glass is manufactured with a certain amount of impurities (air bubbles or trace minerals), this change significantly the behavior of the light beam by changing the spread in the material and avoiding that is the same.

Each process of signal propagation has a normal proceeds to go to a medium different then deviates in an optical path unchecked, this depends on the thickness of the reflective material, practically;
To prove that the use of a third reflecting means is a viable option, practically determined refractive value, this was due to the Brewster angle calculation which mentions a reflection angle where added a transmission angle form an angle of $\pi / 2$, using the air as a means of propagation of light, the refractive index by Snell's law is 1, then through this law we can calculate the index of refraction of the material missing.

Snell's law says that if we have an incident light beam $S$ (Figure 24.) This other than being reflected transmits information via reflection when the ray passes from one medium to another undergoes a change of speed and direction; this depends on the refractive index of each of the mass and the relationship between them, if the index of the second medium is higher than the first target will be seen at a smaller angle with respect to normal, and vice versa if the index is smaller.

To calculate the refractive index practically was employed the law of Snell [10].

$$n_1 \sin \theta = n_2 \sin \alpha$$  \hspace{1cm} (18)

Where:
- $n_1$ = refractive index of medium 1
- $n_2$ = index of refraction of medium 2
- $\theta$ = angle of incidence with respect to the normal
- $\alpha$ = transmission angle with respect to the normal

For Brewster angle we have:

$$\theta + \alpha = 90^\circ$$  \hspace{1cm} (19)

Solving for $\alpha$:

$$\alpha = 90^\circ - \theta$$  \hspace{1cm} (20)

Substituting $\alpha$ in equation (18):

$$n_1 \sin \theta = n_2 \sin(90^\circ - \theta)$$  \hspace{1cm} (21)

From the properties of sine, we have:

$$\sin(a - b) = \sin a \cos b - \sin b \cos a$$  \hspace{1cm} (22)

Applying equation (22) into (21) yields:

$$n_1 \sin \theta = n_2 \cos \theta$$  \hspace{1cm} (23)

Solving $N_2$ of the equation (23) we obtain:

$$n_2 = n_1 \frac{\sin \theta}{\cos \theta}$$  \hspace{1cm} (24)

Getting by trigonometric properties the following result:

$$n_2 = n_1 \tan \theta$$  \hspace{1cm} (25)

Practically values common glass was obtained. See table 1.

Table 1: Result of Brewster angle measurements

<table>
<thead>
<tr>
<th>ángulo inicial (°)</th>
<th>ángulo final (°)</th>
<th>índice de refracción</th>
</tr>
</thead>
<tbody>
<tr>
<td>181</td>
<td>237</td>
<td>1.482560968</td>
</tr>
<tr>
<td>181</td>
<td>236</td>
<td>1.428148007</td>
</tr>
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<tr>
<td>181</td>
<td>236.5</td>
<td>1.455009029</td>
</tr>
<tr>
<td>181</td>
<td>n=</td>
<td>1.44</td>
</tr>
</tbody>
</table>

In practice a refractive index of 1.44 compared to the literature is 1.52, to perform these tests in a proposed new material gives us a value of 2. A glass thickness of 0.85 mm was used for this new material, and the third means comprises a combined application of bi SnO2 distilled water, both in the form of gas, applied to the second
medium at a temperature of 475 °C with a flow of 25 gpm, this creates a very thin layer in the second material, which increases the refractive index of 1.50. Having the third medium thick by tenths or thousandths of a millimeter, internal refractions are almost nil. This is testable with the graph of Figure 24 which are projections luminescence measurements in a changing color sphere. The sphere changes color every 10 seconds, allowing luminescence readings obtained of device.

This amount of light is measured in lux (intensity per square meter on a surface), as can be seen from the graph, each color results in a number of different lux this for just one view of the model of a sphere projected on the entire screen of display. This is an average of the number of colors to represent, can give a range from the least to the most significant, from 52 to 72.67, approximately; in order to know what was the most amount of light intensity that can deliver; the device was held together the amount of information sent in the projection of the four views on the changing sphere model which reported measurements of the graph of Figure 25.

To this one has to the total amount of information of each of the projections drastically low compared to that of a view; since in such case is the investigation of all views. For the light intensity for each model view each value should be divided by the number of projections of the system, in this case four, therefore obtain a different graphic from the previous, as the one shown in Figure 26.

Because the projection is uniform, that is, and is an area of one color. Once the system is the model in the real world different samples of reflection are measured, both made of a layer spread over, as with a simple glass sample, as seen in the graph in Figure 27.

Figure 26. Valor luminescence for single view.

To this one has to the total amount of information of each of the projections drastically low compared to that of a view; since in such case is the investigation of all views. For the light intensity for each model view each value should be divided by the number of projections of the system, in this case four, therefore obtain a different graphic from the previous, as the one shown in Figure 26.

Figure 27 Results of tests on three samples with and without application of the third transmission layer.

From the graph we see a significant difference in values between any of the samples with the third layer and without it, this means that the third layer meets its operating principle, which is to reflect more of information or less information is transmitted to second means. You may also notice that the amount of information received is about half or even less than that sent by the system.

7. Conclusions

VRML allows us to implement all kinds of algorithm and models play on mobile devices.
In optics the projection angle must be defined with respect to the user point of view, this is important as it could reach other design, then according to this principle, all calculations for a specific angle is changed considering that value in the calculations, since so far only depend on one variable.

The projective tests yielded satisfactory results because the user perceives that there is a virtual object projected as a reality, the algorithm needs individual controls for the rotation of the virtual object, as this increased resolution is obtained while you rotate the virtual object. As regards the examination and adaptation to any type of world or virtual object, there is no disadvantage.

Light measurements resulted expected both mathematically and experimentally, which is thought to have increased uptake or information to the viewer; material gives a great advantage to be able to perform the projection in an environment with lighting. It is possible deployment a solution for a mobile device.

Dimensional models are Genera, through mathematical calculations that can be displayed as holograms, more than holograms is the perception that the object is suspended in the air, many people call holography, but as such is an optical effect which is part holography, called perception of three-dimensionality.

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10. References