Abstract: This paper identifies the comprising shape priors in image segmentation has become a key problem in computer vision. Most of the researchers were focuses number of existing works and those were limited to a linearized shape space with small distort modes around a mean shape. These approaches are relevant only when the learning set is composed of very similar shapes. Also, there is no guarantee on the visual quality of the resulting shapes. We introduce a non-linear shape prior for the deformable model framework that we learn from a set of shape samples using recent manifold learning techniques. Our method computes a Delaunay triangulation of the reduced space, considered as Euclidean, and uses the resulting space partition to identify the closest neighbors of any given shape based on its Nystrom extension. First, we propose a solution to the pre-image problem and define the projection of a shape onto the manifold. Based on nearest neighbors for the Diffusion distance, we then describe a variant framework for manifold de-noising. Finally, we introduce a shape prior term for the distort framework through a non-linear energy term designed to attract a shape towards the manifold at given constant implanting.

I. INTRODUCTION

Vision is the most advanced sense among the five senses of human beings, and plays the most important role in human perception. Although there is no clear distinction among image processing, image analysis, and computer vision, usually they are considered as hierarchies in the processing continuum. The low-level processing, which involves primitive operations such as noise filtering, contrast enhancement, and image sharpening, is considered as image processing. Note both its inputs and outputs are images. The mid-level processing, which involves segmentation and pattern classification, is considered as image analysis or image understanding.

The challenge is that images are usually corrupted by several artifacts, such as missing or occluded parts, noise, intensity in-homogeneity or non-uniformity, and partial volume averaging effect. When dealing with complex images, some prior shape knowledge may be necessary to clear up the segmentation process.

A significant work in this direction is the active shape model of Cootes et.al, [1]. A principal component analysis (PCA) on the position of some landmark points placed in a coherent way on all the training contours is used to reduce the number of degrees of freedom to the principal modes of variation. Although successfully applied to various types of shapes (hands, faces, and organs), the trust on a parameterized representation and the manual positioning of the landmarks, particularly tedious in 3D images, seriously limits its applicability.

Leventon et.al, [2] beat these limitations by calculating parameterization-independent shape statistics within the level set representation [3]. Basically, they perform a PCA on the signed distance functions of the training shapes, and the resulting statistical model is integrated into a geodesic active contour framework. The evolution equation contains a term which attracts the model toward
an optimal prior shape as a combination of the mean shape and of the principal modes of variation. Several improvements to this approach have been proposed [4, 5], and in particular an refined integration of the statistical shape model into a unique MAP Bayesian optimization. Let us also mention another neat Bayesian prior shape formulation, based on a B-spline representation, proposed by Cremers et.al, [6].

Performing PCA on distance functions might be tough since they do not define a vector space. To cope with this, Charpiat et.al, [7] proposed shape statistics based on differentiable approximations of the Hausdorff distance. However, their work is limited to a linearized shape space with small deformation modes around a mean shape. Such an approach is relevant only when the learning set is composed of very similar shapes. Lastly, note that the method presented in this paper is different and far superior to the preliminary work introduced in [8].

**Background of our approach**

In this paper, we deviate from the small deformation assumption and introduce a new deformable model framework that integrates more general non-linear shape priors. We model a category of shapes as a smooth finite dimensional sub-manifold of the infinite-dimensional shape space, termed the shape prior manifold. This manifold which cannot be represented explicitly is approximated from a collection of shape samples using a recent manifold learning technique called diffusion maps [9,10]. Manifold learning, which is already an established tool in object recognition and image classification, has been recently applied to shape analysis [11]. Yet, to our knowledge, such techniques have not been used in the context of image segmentation with shape priors.

Diffusion maps generate a mapping, called an embedding, from the original shape space into a low-dimensional space. Well, this mapping is an isometric from the original shape space, equipped with a diffusion distance, into a low-dimensional Euclidean space [9]. In this paper, we exploit the isometric mapping and the Euclidean nature of the reduced space to design our variant shape prior framework. We propose to introduce a shape prior term for the deformable framework through a non-linear energy term designed to attract a shape towards its projection on to the manifold. Doing so requires being able to estimate the manifold between training samples and to compute the projection of a shape onto the manifold. Unfortunately, diffusion maps do not give access to such tools.

Our contribution lies in three aspects. First, we propose a solution to the estimation of the manifold between training samples. We define a projection operator onto the manifold based on: 1) Nystrom extensions [12] which provide a effectual framework for extending embedding coordinates (in the shape prior manifold) to the full infinite dimensional shape space, 2) a Delaunay partitioning of the reduced space to identify the closest neighbors (in the training set) of any shape in the original infinite dimensional shape space. In light of this, we then describe a variant framework for manifold denoising, thereby decreasing the negative impact of outliers onto our variant shape framework. Finally, we describe our shape prior term integrated in the deformable model framework through a nonlinear energy term designed to attract a shape towards the manifold at given constant embedding.

**II. SYSTEM DESIGN MODEL**

**A. Learning the Shape Prior Manifold**

In the sequel, we define a shape as a simple compact (i.e. closed and non-intersecting) surface, and we denote by \( S \) the (infinite-dimensional) space of such shapes. We make the assumption that a category of shapes, i.e. the set of shapes that can be identified with a common anatomical structure, e.g. left or right ventricle, hippocampus nucleus, be modeled as a finite-
Dimensional manifold, termed the shape prior manifold. Dimensionality reduction, i.e. the process of recovering the underlying low dimensional structure of a manifold M embedded into a higher-dimensional space, has enjoyed renewed interest over the past years. Among the most recent and popular techniques are the Locally Linear Embedding (LLE), Laplacian eigen maps, Diffusion maps.

In this work, we learn the shape prior manifold using Diffusion maps. For the sake of clarity, we present the mathematical formulation for data living in \( \mathbb{R}^n \). An extension to infinite-dimensional shape manifolds is straightforward.

### Manifold Learning and Diffusion Maps

Let M be a manifold of dimension m lying in \( \mathbb{R}^n \) (\( m < n \)). Diffusion maps rely on discrete approximations of the Laplace-Beltrami operator \( \Delta M \) defined on the manifold M to generate a mapping (called an embedding) \( f : M \rightarrow \mathbb{R}^m \) such that if two points x and z are close, so are \( f(x) \) and \( f(z) \). The optimal mapping is given by the eigen-functions of the Laplace-Beltrami operator corresponding to the m smallest non-zero eigen values, where m is the target dimension. Note that the latter dimension can either be known a priori or be inferred from the profile of the eigen spectrum. In practice, a discrete counterpart to this continuous formulation must be used since we only have accesses to a distinct and finite set.

### Distance in the Shape Space

The approximation of the Laplace-Beltrami operator requires the choice of a distance between shapes. Many different definitions of the distance between two shapes have been proposed in the computer vision literature but there is no agreement on the correct way of measuring shape similarity. The definitions used in the experiments are based on the representation of a surface \( S \) in the Euclidean embedding space \( \mathbb{R}^3 \) by its signed distance function. In this context, we define the distance between two shapes to be the Sobolev \( W^{1,2} \)-norm of the difference between their signed distance functions.

\[
d_{w^{1,2}}(S_1, S_2)^2 = \left\| \overline{D}_{S_1} - \overline{D}_{S_2} \right\|_{L^2(\omega, \mathbb{R})}^2 + \left\| \nabla \overline{D}_{S_1} - \nabla \overline{D}_{S_2} \right\|_{L^2(\omega, \mathbb{R}^3)}^2,
\]

Where \( \overline{D}_{S_i} \) denotes the signed distance function of shape \( S_i \) (\( i = 1, 2 \)), and \( \nabla \overline{D}_{S} \) its gradient. Note that to define a distance between shapes that is invariant to rigid displacements (rotations and translations), we first align the shapes using their principal consequences before computing distances. Note also that the proposed method is apparently not limited to a particular choice of distance.

### B. Image Segmentation using the Shape Prior Manifold

In this section, we propose to use the embedding to carefully design a shape prior term merged into a deformable model framework for image segmentation.

### Image Segmentation as a variant Problem

Without loss of generality, we cast the segmentation problem as a variant one, where the objective is to find a surface \( S \) minimizing a global energy functional \( E^{ac} \). Depending on the segmentation task and the available information, the energy functional \( E^{ac} \) can take on different, more or less complex, forms, but, in general \( E^{ac} \) can be written as a combination of image terms, regularization terms and smoothness constraints. Directly finding the global minimum of \( E^{ac} \) is usually impossible and one often has to resort to a suboptimal gradient-descent strategy starting from a guess \( S_0 \). That is we assume that the image segmentation problem amounts to solving the following evolution problem: find the active contour \( S : \tau \in \mathbb{R}^+ \rightarrow S(\tau) \in \mathbb{S} \) such that

\[
S(0) = S_0, \quad \frac{dS}{d\tau} = -\nabla E^{ac}.
\]

### Designing and Integrating the Shape Prior Term


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We define a shape prior functional $E^{sp}$ designed to attract any given shape $S$ towards the shape prior manifold. Unfortunately, Diffusion maps do not give access to an explicit projection operator onto the reduced manifold. To ease this problem, we exploit the Euclidean nature of the reduced space by computing a Delaunay triangulation in $\mathbb{R}^m$ of the training data. The space partition is then used to identify the $m+1$ closest neighbors (in the training set $\mathcal{G}$) of the shape $S$ in $\mathbb{S}$ by computing its embedding coordinates $\phi(S)$ and finding the corresponding Delaunay triangle formed by $m+1$ vertices in $\mathbb{R}^m$. By doing so, we identify the $m+1$ closest neighbors $N = (S_0, ..., S_m)$ of $S$ in $\mathbb{S}$ for the Diffusion metric [10]. This neighborhood $N$ will then be used to attract $S$ towards the manifold.

To this end, we compute the barycentric coordinates $\Theta = (\theta_0, \cdots, \theta_m)$ of the shape $S$ in the reduced space $\mathbb{R}^m$ and define the shape prior functional in $\mathbb{S}$:

$$E^{sp}_{N,\Theta} = \sum_{i=0}^{m} \theta_i d^2(S_i, S)$$

with

$$\Phi(S) = \sum \theta_i \Phi(S_i), \theta_i \geq 0, \sum \theta_i = 1$$

designed to attract the shape $S$ towards a weighted mean shape that interpolates between the $m+1$ samples $S_i \in N$.

Minimization of the energy $E^{sp}_{N,\Theta}$ by gradient descent might change the embedding coordinates $\Phi_i(S_i)$ of the evolving shape $S$. Therefore, denoting by $\mathbb{S}_x = \theta_i^{-1}(x)$ the $x$-level set in $S$ of the embedding $\Phi_i$ (note that $\mathbb{S}_x$ has co-dimension $m$), we define the shape prior term $\bar{v}^{sp}$ as the projection of the velocity field $\bar{v} = -\nabla E^{sp}_{N,\Theta}$ onto the tangent space $T_{\Phi_i(x)}$ of $S$. The $T_{\Phi_i(x)}$ can be expressed by $m$ simple orthogonality conditions in the tangent space $T_{\Phi_i(x)}$ of $\mathbb{S}$ at $S$:

$$T_{\Phi_i(x)} = \{ \bar{v} \in T_i(S) : \forall k = 1, ..., m \sum_{S \in \mathbb{S}} \langle \nabla^{sp}(S, S_j) | \bar{v} \rangle_{L^2} \Psi_k(S_j) = 0 \}$$

Where $\langle | \rangle_{L^2}$ corresponds to the $L^2$-dot product in the tangent shape space $T_S(S)$. Projection of the velocity field $-\nabla E^{sp}_{N,\Theta}$ onto $T_{\Phi_i(x)}$ can then be achieved using the orthogonalization Gram-Schmidt process.

Finally, the general deformable model framework corresponds to solving the following evolution problem:

$$\frac{dS}{d\tau} = -\nabla E^{ac} + \alpha \bar{v}^{sp}$$

Where $\alpha$ is a weighting parameter. Note that at each step of the evolution, we have to align the shape with the training samples using the principal moments before computing its embedding and during the shape prior term $\bar{v}^{sp}$.

### III. SIMULATION AND EXPERIMENTAL RESULTS

**Experiment-1:**
Consider the following image training data set:

![Image](image1.png)

Consider following corrupted image as test image:

![Image](image2.png)
Then the segmentation process as follows:

Step-1: Finding the Shape-prior for image training set: In this shape can be defined as a compact (i.e. closed and non-intersecting) surface with assumption that all images belongs to same category of structures. After applying this to the above training data set, we create a database of shapes, termed as shape-prior manifold as shown below:

Step-2: Find the Shape-prior for test image:

Step-3: Compare the shape-prior of the test image training data set:

Step-4: Do segmentation using diffusion map to find the contour of the test image shown by red color:

Experiment-2:
Consider the following image training data set:

Consider following corrupted image as test image:

Then the segmentation process as follows:
Step-1: Find the Shape-prior for image training set: In this shape can be defined as a compact (i.e. closed and non-intersecting) surface with assumption that all images belongs to same category of structures. After applying this to the above training data set, we create a database of shapes, termed as shape-prior manifold as shown below:
IV. CONCLUSION

We presented a new projection operator onto a manifold based on the Nystrom extension and a Delaunay partitioning of the reduced space. We then provided a variant solution for manifold de-noising. Finally, we expressed a new energy term designed to attract a shape towards the manifold at given constant embedding. We demonstrated the strength of our approach by applying these ideas in different experiments either with synthetic or real data, including in segmentation tasks.

V. REFERENCES


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