REALISING STATE MODEL FOR AIR SEPARATION UNIT USING SUBSPACE IDENTIFICATION METHOD

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Abstract:
In this paper the state model formation using subspace identification method is discussed and implemented for the real time distillation column used in air separation unit. The algorithm used is numerical method for state space subspace identification (N4SID), the data is processed for modelling and results are discussed.

Keywords: Subspace Identification, N4SID, MIMO, ASU.

I. INTRODUCTION

The state model approach is an economical way to model any non-linear multi-input-multi-output (MIMO) systems which employs large industrial process and exhibiting a pure dynamic behaviour. For state space model the first or second order differential equation of the system is required, this can be achieved in two ways, the mechanical and electrical equivalent transfer functions can be used to model or in a better way; the input output data can be used as the modelling parameters using the N4SID algorithm of subspace identification method. In the following chapters a detail discussion is made about system identification, subspace identification method, N4SID algorithm and exploiting the algorithm in order to work out with the available data.

1.1 State Space Model

Models in this paper are lumped, discrete time, linear, time-invariant, state space models. From the number of epithets used, this might seem like a highly restricted class of models (especially the fact they are linear), but, surprisingly enough, many industrial processes can be described very accurately by this type of models, especially locally in the neighbourhood of a working point. Moreover, there are a large number of control system design tools available to build controllers for such systems and models. Mathematically, these models are described by the following set of difference equations:

\[
\begin{align*}
    x_{k+1} &= Ax_k + Bu_k + w_k, \\
    y_k &= Cx_k + Du_k + v_k,
\end{align*}
\]

With

\[
E\left[ \begin{pmatrix} w_p^T \\ y_p^T \end{pmatrix} \right] = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \delta_{pq} \geq 0
\]

Figure 1 A graphical representation of the system
II. SYSTEM IDENTIFICATION

2.1 Sub-space identification:
In system identification methods the subspace identification is proven to be more optimum and flexible for nonlinear data and nonlinear modelling. The goal of this Section is to provide a verbal description of the main principles on which subspace identification algorithms are based. The mathematical derivations will be elaborated on in the next sections. The state sequence of the dynamical system is determined first, directly from input/output observations, without knowing the model. That this is possible for the model class I is one of the main contributions of subspace algorithms, as compared to “classical” approaches that are based on an input-output framework. The difference is illustrated in Figure 2. So an important achievement of the research in subspace identification was to demonstrate how the Kalman filter states can be obtained directly from input-output data using linear algebra tools (QR and singular value decomposition) without knowing the mathematical model. An important consequence is that, once these states are known, the identification problem becomes a linear least squares problem in the unknown system matrices and the process and measurement noise covariance matrices follow from the least squares residuals, as is easy to see from Equations (1):

![Figure 2: Subspace Algorithms](image)

Subspace identification aims at constructing state space models from input-output data. The left hand side shows the subspace identification approach: first the (Kalman filter) states are estimated directly (either implicitly or explicitly) from input-output data, then the system matrices can be obtained. The right hand side is the classical approach: first obtain the system matrices, then estimate the states.

2.1 Kalman Filter and N4SID:
The Kalman filter is a set of recursion equations for determining the optimal estimates of the state vector at given information available at time t, It. The filter consists of two sets of equations:

1. Prediction equations
2. Updating equations

To describe the filter, let

\[ a_t = E[a_t|I_t] = \text{optimal estimator of } a_t \text{ based on } I_t \]
\[ P_t = E[(a_t - a_t)(a_t - a_t)\text{0}|I_t] = \text{MSE matrix of } a_t \]

2.2.1 Prediction Equation:
Given \( a_{t-1} \) and \( P_{t-1} \) at time \( t - 1 \), the optimal predictor of \( a_t \) and its associated MSE matrix is

\[
\begin{align*}
    a_t|t-1 & = E[a_t|I_t] = T_t a_{t-1} + c_t \\
    P_t|t-1 & = E[(a_t - a_{t-1})(a_t - a_{t-1})\text{0}|I_{t-1}] \\
    & = T_t P_{t-1} T_0 + R_t Q_t R_0 t
\end{align*}
\]

The corresponding optimal predictor of \( y_t \) give information at \( t - 1 \) is

\[
y_t|t-1 = Z_t a_{t-1} + d_t
\]

The prediction error and its MSE matrix are

\[
\begin{align*}
    v_t & = y_t - y_t|t-1 = y_t - Z_t a_{t-1} - d_t \\
    & = Z_t(a_t - a_{t-1}) + c_t
\end{align*}
\]

\[
E[v_t v_0^t] = F_t = Z_t P_t|t-1 Z_0 + H_t
\]

These are the components that are required to form the prediction error decomposition of the log-likelihood function.

2.2.2 Updating Equation:
When new observations \( y_t \) become available, the optimal predictor \( a_{t|t-1} \) and its MSE matrix are updated using:

\[
\begin{align*}
    a_t = a_{t|t-1} + P_{t|t-1} Z_t F^{-1} (y_t - Z_t a_{t|t-1} - d_t) \\
    & = a_{t|t-1} + P_{t|t-1} Z_0 F^{-1} - I_{t-1} v_t
\end{align*}
\]
\[ P_t = P_{t-1} - Pt|t-1ZtF^{-1}t ZtPt|t-1 \]

The value at is referred to as the filtered estimate of \( \alpha_t \) and \( P_t \) is the MSE matrix of this estimate. It is the optimal estimate of \( \alpha_t \) given information available at time \( t \).

2.2.3 Prediction Error Decomposition

Let \( \theta \) denote the parameters of the state space model. These parameters are embedded in the system matrices. For the state space model with a fixed value of \( \theta \), the Kalman Filter produces the prediction errors, \( vt(\theta) \), and the prediction error variances, \( Ft(\theta) \), from the prediction equations. The prediction error decomposition of the log-likelihood function follows immediately:

\[
\ln L(\theta|y) = -\frac{NT}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=1}^{T} \ln|F_t(\theta)| - \frac{1}{2} \sum_{t=1}^{T} v_t^\top(\theta)F_t^{-1}(\theta)v_t(\theta)
\]

Incorporating all the above equations Kalman Filter coefficients are updated and hence the state space model is achieved. A detailed discussion is given in [6] regarding Kalman Filter and state model. The next step is to identify the state model of ASU with suitable data samples, and the model accuracy.

III. MODELLING OF ASU

Before implementing the subspace identification method it is necessary to preprocess the data for any removal of unwanted saturation or irrelevant relations between the input vector and the output vector of the given data sample. In the given data we consider thousand samples as the sample space for modelling, each data input and will have some correlation with the data output, the sample space which is having more correlation is only considered. The input output data to be used are as follows:

<table>
<thead>
<tr>
<th>TABLE 3.1</th>
<th>INPUT OUTPUT DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>Output</td>
</tr>
<tr>
<td>Liquid air</td>
<td>O₂ Purity Analyser</td>
</tr>
<tr>
<td>Air input</td>
<td></td>
</tr>
<tr>
<td>Air input to turbine</td>
<td></td>
</tr>
<tr>
<td>Oxygen Flow</td>
<td></td>
</tr>
</tbody>
</table>

3.1 Data analysis:

All inputs in the data have been denoted as 'zero order hold' ('zoh'), i.e. they are assumed to be piecewise constant over the sampling interval. If the input is a sampled continuous signal and you plan to build or convert to time continuous models, it is recommended to mark the InterSample property as 'First order hold': Data.int = 'foh' or Data.int = {'foh', 'foh', ...} for multi-input signals. You may mix 'zoh' with 'foh' for the different inputs in the latter case. All inputs and outputs are not zero mean. It it generally recommended to remove the means except in the following cases:

1. The signals are measured relative to a level that corresponds to a physical equilibrium. This could e.g. be the case if step responses are recorded from an equilibrium point. In this case, it is advisable to remove the equilibrium values rather than data means.

2. There is an integrator in the system, and the input and output levels are essential to describe the effect of the integration.

There is a very strong indication of feedback in the data. One should be careful when interpreting the results of SPA and also interpret the results of output error models with care. With feedback in data, it is recommended to use estimate a model with large enough disturbance model.

3.2 Correlation factor

Prior to fix the sample space it is necessary to identify the correlation factors between the individual inputs to the output as detailed in table 3.2

<table>
<thead>
<tr>
<th>TABLE 3.2</th>
<th>CORRELATION FACTORS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid air</td>
<td>0.0649</td>
</tr>
<tr>
<td>Air input</td>
<td>-0.0214</td>
</tr>
<tr>
<td>Air input to turbine</td>
<td>0.0984</td>
</tr>
<tr>
<td>Oxygen Flow</td>
<td>0.2468</td>
</tr>
<tr>
<td>Nitrogen flow</td>
<td>0.1060</td>
</tr>
</tbody>
</table>
3.3 Smoothening the data samples

As per the data analysis it will be easy to model if the given samples of data are smoothened to remove any unwanted non-linearity and irrelevant correlations between the input and output sample space.

3.3.1 Input variable smoothening:

![Figure 3 Liquid air input](image)

![Figure 4 Nearest fitting for Liquid air input](image)

![Figure 5 Air input](image)

![Figure 6 Nearest fitting](image)

![Figure 7 Air inlet to turbine flow rate](image)

![Figure 8 Nearest fitting](image)

![Figure 9 Oxygen flow rate](image)

![Figure 10 Nearest fit](image)

IV. RESULTS AND DISCUSSIONS

With the data pre-processing the modelling is performed using the subspace identification method and following model results are achieved:
The real-time model for Distillation column is estimated using state space analysis and from the A, B, C & D parameters the model is described into a discrete time LTI system. It is observed that the system approaches nearly to a real-time system with 96.7% results matching with the desired outputs. More detailed results are discussed in [3],[4] and [5].

V. CONCLUSION
In system identification methods the subspace identification is proven to be more optimum and flexible for nonlinear data and nonlinear modelling. However a detailed study is to be made for further optimization of modelling and easy calculations with different orders of system. The method to be discussed for further analysis is ARMA modelling method which finds an optimum way to estimate a non-linear system to an equivalent linear system.

VI. REFERENCES
[5]. Peter Van Overschee Bart De Moor ‘subspace identification for linear systems’

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