An Application of Similarity of Fuzzy Soft Sets in Decision Making

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Abstract

Application of fuzzy soft sets in decision making problems is a recent topic to solve real life problems in an imprecise environment. In this paper, we have applied the notion of similarity between two fuzzy soft sets to obtain the solution of a decision problem in an imprecise environment.

1. Introduction

In many complicated problems arising in the fields of engineering, social science, economics, medical science etc involving uncertainties, classical methods are found to be inadequate in recent times. Molodtsov [3] pointed out that the important existing theories viz. Probability Theory, Fuzzy Set Theory, Intuitionistic Fuzzy Set Theory, Rough Set Theory etc. which can be considered as mathematical tools for dealing with uncertainties, have their own difficulties. He further pointed out that the reason for these difficulties is, possibly, the inadequacy of the parameterization tool of the theory. In 1999 he initiated the novel concept of Soft Set as a new mathematical tool for dealing with uncertainties. Soft Set Theory, initiated by Molodtsov [3], is free of the difficulties present in these theories. In recent times, researches have contributed a lot towards fuzzification of Soft Set Theory. Maji et al. [2] introduced the concept of Fuzzy Soft Set and some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, DeMorgan Law etc. These results were further revised and improved by Ahmad and Kharal [1]. Similarity measures have extensive application in pattern recognition, region extraction, coding theory, image processing and in many other areas. Similarity of two fuzzy soft sets has been studied by Majumder and Samanta in [4] and application of similarity between generalized fuzzy soft sets has been studied by them in [5]. In this paper, we are using the notion of similarity of fuzzy soft sets initiated by Majumder and Samanta in [4] and an attempt has been made to apply the same in solving a decision problem.

2. Preliminaries

In this section, we first recall the basic definitions related to fuzzy soft sets which would be used in the sequel.

2.1. Soft Set [3]
A pair \((F, E)\) is called a soft set (over \(U\)) if and only if \(F\) is a mapping of \(E\) into the set of all subsets of the set \(U\). In other words, the soft set is a parameterized family of subsets of the set \(U\). Every set \(E \in F\) may be considered as the set of \(E\)-elements of the soft set \((F, E)\), or as the set of \(E\)-approximate elements of the soft set.

2.2. Fuzzy Soft Set [2]
A pair \((F, A)\) is called a fuzzy soft set over \(U\) where \(F : A \mapsto P(U)\) is a mapping from \(A\) into \(P(U)\).

2.3. Fuzzy Soft Class [1]
Let \(U\) be a universe and \(E\) a set of attributes. Then the pair \((U, E)\) denotes the collection of all fuzzy soft sets on \(U\) with attributes from \(E\) and is called a fuzzy soft class.

2.4. Fuzzy Soft Null Set [2]
A soft set \((F, A)\) over \(U\) is said to be null fuzzy soft set denoted by \(\varnothing\) if \(\forall \epsilon \in A, F(\epsilon)\) is the null fuzzy set \(\varnothing\) of \(U\) where \(\varnothing(x) = 0\) for all \(x \in U\).

2.5. Fuzzy Soft Absolute Set [2]
A soft set \((F, A)\) over \(U\) is said to be absolute fuzzy soft set denoted by \(\tilde{A}\) if \(\forall \epsilon \in A, F(\epsilon)\) is the absolute fuzzy set \(\tilde{A}\) of \(U\) where \(\tilde{A}(x) = 1\) for all \(x \in U\).

2.6. Fuzzy Soft Subset [2]
For two fuzzy soft sets \((F, A)\) and \((G, B)\) in a fuzzy soft class \((U, E)\), we say that \((F, A)\) is a fuzzy soft subset of \((G, B)\), if

(i) \(A \subseteq B\)
(ii) For all \( e \in A \), \( F(e) \subseteq G(e) \) and is written as 
\((F, A) \subseteq (G, B)\).

Union of two fuzzy soft sets \((F, A)\) and \((G, B)\) in a soft class \((U, E)\) is a fuzzy soft set \((H, C)\) where \(C = A \cup B\) and \(\forall e \in C\),
\[
H(e) = \begin{cases} 
F(e), & \text{if } e \in A - B \\
G(e), & \text{if } e \in B - A \\
F(e) \cup G(e), & \text{if } e \in A \cap B
\end{cases}
\]
And is written as \((F, A) \cup (G, B) = (H, C)\).

Intersection of two fuzzy soft sets \((F, A)\) and \((G, B)\) in a soft class \((U, E)\) is a fuzzy soft set \((H, C)\) where \(C = A \cap B\) and \(\forall e \in C\), \(H(e) = F(e)\) or \(G(e)\) (as both are same fuzzy set) and is written as \((F, A) \cap (G, B) = (H, C)\).

Ahmad and Kharal [1] pointed out that generally \(F(e)\) or \(G(e)\) may not be identical. Moreover in order to avoid the degenerate case, he proposed that \(A \cap B\) must be non-empty and thus revised the above definition as follows -

2.9. Intersection of Fuzzy Soft Sets Redefined [1]
Let \((F, A)\) and \((G, B)\) be two fuzzy soft sets in a soft class \((U, E)\) with \(A \cap B \neq \emptyset\). Then Intersection of two fuzzy soft sets \((F, A)\) and \((G, B)\) in a soft class \((U, E)\) is a fuzzy soft set \((H, C)\) where \(C = A \cap B\) and \(\forall e \in C\), \(H(e) = F(e)\) or \(G(e)\) (as both are same fuzzy set) and is written as \((F, A) \cap (G, B) = (H, C)\).

Let \(U = \{x_1, x_2, x_3, \ldots, x_m\}\) be the universal set of elements and \(E = \{e_1, e_2, e_3, \ldots, e_p\}\) be the universal set of parameters. Let \(\hat{F} = (F, E)\) and \(\hat{G} = (G, E)\) be two fuzzy soft sets over \((U, E)\). Then \(\hat{F} = \{F(e_j) \in P(U) : e_j \in E\}\) and \(\hat{G} = \{G(e_j) \in P(U) : e_j \in E\}\), where \(F(e_j)\) is called the \(e_j\) -th approximation of \(\hat{F}\) and \(G(e_j)\) is called the \(e_j\) -th approximation of \(\hat{G}\). \(P(U)\) be the collection of all fuzzy subsets of \(U\). Let \(M[\hat{F}, \hat{G}]\) indicates the similarity between the fuzzy soft sets \(\hat{F}\) and \(\hat{G}\). To find the similarity between \(\hat{F}\) and \(\hat{G}\), first we have to find the similarity between their \(e\) - approximations. Let \(M_j[\hat{F}, \hat{G}]\) denote the similarity between the two \(e_j\) – th approximations \(F(e_j)\) and \(G(e_j)\).

Then we define
\[
M_j[\hat{F}, \hat{G}] = \sum_{i=1}^{m} \left[ \frac{F_{ij} \land G_{ij}}{\sum_{j=1}^{m} F_{ij} \lor G_{ij}} \right],
\]
where \(F_{ij} = F(e_j \cap x_i) \subseteq I\) and \(G_{ij} = G(e_j \cap x_i) \subseteq I\).

Then \(M_{F,G} = \max_j M_j[\hat{F}, \hat{G}]\), \(j = 1, 2, \ldots, n\).

3. Application Of Similarity In Decision Making

Analogous to Majumder and Samanta [5], we would call the fuzzy soft sets \((F, E)\) and \((G, E)\) in the fuzzy soft class \((U, E)\) to be significantly similar if
\[
M((F, E), (G, E)) > \frac{1}{2}
\]
Suppose the authority of an institution wants to give award to the performing students in an academic year. We assume that after some screening rounds, three students are available for the award. Let our universal set contain only two elements ‘yes (y)’ and ‘no (n)’, i.e. \(U = \{y, n\}\). Here the set of parameters \(E\) is the set of certain approximations determined by the authority. Let \(E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}\), where \(e_1\) = sincerity, \(e_2\) = extracurricular activity, \(e_3\) = pleasing personality, \(e_4\) = good moral character, \(e_5\) = sports activity, \(e_6\) = communication skills, \(e_7\) = examination performance. Our model fuzzy soft set for the performing students \((F, E)\) is given in Table 1. Similarly, we construct the fuzzy soft sets for the three students under consideration as given in Table 2, 3 and 4.

Table 1: Model fuzzy soft set for performing student

<table>
<thead>
<tr>
<th>((F, E))</th>
<th>(e_1)</th>
<th>(e_2)</th>
<th>(e_3)</th>
<th>(e_4)</th>
<th>(e_5)</th>
<th>(e_6)</th>
<th>(e_7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y, n)</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(y, n)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: Fuzzy Soft Set for first student under consideration:

<table>
<thead>
<tr>
<th>((F, E))</th>
<th>(e_1)</th>
<th>(e_2)</th>
<th>(e_3)</th>
<th>(e_4)</th>
<th>(e_5)</th>
<th>(e_6)</th>
<th>(e_7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y, n)</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(y, n)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

4. Conclusion

In this paper, we discussed the theory of fuzzy soft sets and their applications in decision making. The results are compared with the results obtained by Majumder and Samanta [5]. The model fuzzy soft set and its approximations are presented in the tables. The similarity measure is calculated and it is found that the fuzzy soft sets are significantly similar if the similarity measure is greater than \(\frac{1}{2}\).
\[(G,E)\quad e_1\quad e_2\quad e_3\quad e_4\quad e_5\quad e_6\quad e_7\]\n\[y\quad 0.3\quad 0.7\quad 0.9\quad 0.1\quad 0\]
\[n\quad 0.6\quad 0.8\quad 0.2\quad 0.3\quad 0.5\quad 0.2\quad 0.1\]

\[M_2(\hat{F},\hat{G}) = \sum_{i=1}^{m} (F_{i2} \land G_{i2}) / \sum_{i=1}^{m} (F_{i2} \lor G_{i2}) = 0+0 / 1+0.8 = 0 / 1.8 = 0\]

Table 3: Fuzzy Soft Set for second student under consideration:

\[(H,E)\quad e_1\quad e_2\quad e_3\quad e_4\quad e_5\quad e_6\quad e_7\]\n\[y\quad 0.1\quad 0.4\quad 0.5\quad 0.2\quad 0.3\quad 0.1\quad 0.7\]
\[n\quad 0.3\quad 0.4\quad 0.2\quad 0.1\quad 0.9\quad 0.7\quad 0.1\]

\[M_3(\hat{F},\hat{G}) = \sum_{i=1}^{m} (F_{i3} \land G_{i3}) / \sum_{i=1}^{m} (F_{i3} \lor G_{i3}) = 0+0.2 / 0.7+1 = 0.2 / 1.7 = 0.12\]

Then
\[\hat{F} = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \]
\[\hat{G} = \begin{pmatrix} 0.3 & 0 & 0.7 & 0.2 & 0.9 & 0.1 & 0 \\ 0.6 & 0.8 & 0.2 & 0.3 & 0.5 & 0.2 & 0.1 \end{pmatrix} \]
\[\hat{H} = \begin{pmatrix} 0.1 & 0.4 & 0.5 & 0.2 & 0.3 & 0.1 & 0.7 \\ 0.3 & 0.4 & 0.2 & 0.1 & 0.9 & 0.7 & 0.1 \end{pmatrix} \]
\[\hat{i} = \begin{pmatrix} 0.3 & 0 & 0.2 & 0.5 & 0.6 & 0.1 & 0 \\ 0.5 & 0.7 & 0.1 & 0.2 & 0.5 & 0.4 & 0.7 \end{pmatrix} \]

We have,
\[M_4(\hat{F},\hat{G}) = \sum_{i=1}^{m} (F_{i4} \land G_{i4}) / \sum_{i=1}^{m} (F_{i4} \lor G_{i4}) = 0.2+0 / 1+0.3 = 0.2 / 1.3 = 0.15\]

\[M_5(\hat{F},\hat{G}) = \sum_{i=1}^{m} (F_{i5} \land G_{i5}) / \sum_{i=1}^{m} (F_{i5} \lor G_{i5}) = 0+0.5 / 0.9+1 = 0.5 / 1.9 = 0.26\]
In view of our discussion we can conclude that the second student may be awarded by the institution authority.

4. Conclusion
We have applied the notion of similarity of fuzzy soft sets in a certain decision making problem. It is hoped that our work would help enhancing this study on fuzzy soft sets.

5. References