Face Representation And Recognition Using Two-Dimensional PCA

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ABSTRACT—In this paper, two-dimensional principal component analysis (2DPCA) is used for image representation and recognition. Compared to 1D PCA, 2DPCA is based on 2D image matrices rather than 1D vectors so the image matrix does not need to be transformed into a vector prior to feature extraction. Instead, an image covariance matrix is constructed directly using the original image matrices, and its eigenvectors are derived for image feature extraction. In order to test the approach, we have used ORL face database images. The recognition rate across all trials was higher using 2DPCA than PCA. The experimental results shows that this approach of extraction of image features is computationally more efficient using 2DPCA than PCA. It is also observed from the results that the recognition rate is high.

Index Terms—Principal Component Analysis (PCA), Eigenfaces, feature extraction, image representation, face recognition.

I. INTRODUCTION

Principal Component Analysis (PCA), also known as Karhunen-Loeve expansion, is a classical feature extraction and data representation technique widely used in the areas of pattern recognition and computer vision. Sirovich and Kirby[1][2] first used PCA to efficiently represent pictures of human faces. They argued that any face image could be reconstructed approximately as a weighted sum of a small collection of images that define a facial basis (eigen images), and a mean image of the face. Within this context, Turk and Pentland[3] presented the well-known Eigen faces method for face recognition in 1991.

Since then, PCA has been widely investigated and has become one of the most successful approaches in face recognition Penev and Sirovich[4] discussed the problem of the dimensionality of the “face space” when eigen faces are used for representation. Zhao and Yang[5] tried to account for the arbitrary effects of illumination in PCA-based vision systems by generating an analytically closed-form formula of the covariance matrix for the case with a special lighting condition and then generalizing to an arbitrary illumination via an illumination equation. However, Wiskott et al[6], pointed out that PCA could not capture even the simplest invariance unless this information is explicitly provided in the training data. They proposed a technique known as elastic bunch graph matching to overcome the weaknesses of PCA.

Recently, two PCA-related methods, independent component analysis (ICA) and kernel principal component analysis (Kernel PCA) have been of wide concern. Bartlett et al.and Draper et al[7], proposed using ICA for face representation and found that it was better than PCA when cosines were used as the similarity measure (however, their performance was not significantly different if the Euclidean distance is used). Yang[8] used Kernel PCA for face feature extraction and recognition and showed that the Kernel Eigenfaces method outperforms the classical Eigenfaces method. However, ICA and Kernel PCA are both computationally more expensive than PCA. The experimental results in showed the ratio of the computation time required by ICA, Kernel PCA, and PCA is, on average, 8.7: 3.2: 1.0.

In the PCA-based face recognition technique, the 2D face image matrices must be previously transformed into 1D image vectors. The resulting image vectors of faces usually lead to a high-dimensional image vector space, where it is difficult to evaluate the covariance matrix accurately due to its large size and the relatively small number of training samples. Fortunately, the eigenvectors (eigenfaces) can be calculated efficiently using the SVD techniques and the process of generating the covariance matrix is actually avoided. However, this does not imply that the eigenvectors can be evaluated
accurately in this way since the eigenvectors are statistically determined by the covariance matrix, no matter what method is adopted for obtaining them.

In this paper, a straightforward image projection technique, called two-dimensional principal component analysis (2DPCA), is developed for image feature extraction. As opposed to conventional PCA, 2DPCA is based on 2D matrices rather than 1D vectors. That is, the image matrix does not need to be previously transformed into a vector. Instead, an image covariance matrix can be constructed directly using the original image matrices. In contrast to the covariance matrix of PCA, the size of the image covariance matrix using 2DPCA is much smaller. As a result, 2DPCA has two important advantages over PCA. First, it is easier to evaluate the covariance matrix accurately. Second, less time is required to determine the corresponding eigenvectors.

The remainder of this paper is organized as follows: In Section II, the idea of the proposed 2DPCA method and its algorithm are described. The image reconstruction method using 2DPCA is developed in Section IV. In Section VIII, experimental results are presented for the ORL, the AR, and the Yale face image databases to demonstrate the effectiveness and robustness of 2DPCA. Finally, conclusions are presented in Section IX.

II. TWO-DIMENSIONAL PRINCIPAL COMPONENT ANALYSIS

Let X denote an n-dimensional unitary column vector. Our idea is to project image A, an m x n random matrix, onto X by the linear transformation

\[ Y = AX \] (1)

Thus, we obtain an m-dimensional projected vector Y, which is called the projected feature vector of image A. How do we determine a good projection vector X? In fact, the total scatter of the projected samples can be introduced to measure the discriminatory power of the projection vector X. The total scatter of the projected samples can be characterized by the trace of the covariance matrix of the projected feature vectors. From this point of view, we adopt the following criterion:

\[ J(X) = \text{tr}(S_r) \] (2)

Where \( S_r \) denotes the covariance matrix of the projected feature vectors of the training samples and \( \text{tr}(S_r) \) denotes the trace of \( S_r \). The physical significance of maximizing the criterion in (2) is to find a projection direction \( X \), onto which all samples are projected, so that the total scatter of the resulting projected samples is maximized. The covariance matrix \( S_r \) can be denoted by

\[ S_r = E(Y-EY)(Y-EY)^T \]
\[ = E[AX-E(AX)][AX-E(AX)]^T \]
\[ = E[(A-EA)X][A-EA)X]^T \]

So,

\[ \text{tr}(S_r) = X^T[E(A-EA)^T(A-EA)]X \] (3)

Let us define the following matrix

\[ G_i = E[(A-EA)^T(A-EA)] \] (4)

The matrix \( G_i \) is called the image covariance (scatter) matrix. It is easy to verify that \( G_i \) is an n x n non-negative definite matrix from its definition. We can evaluate \( G_i \) directly using the training image samples. Suppose that there are M training images samples in total, the jth training image is denoted by an m x n matrix \( A_j \) (j = 1, 2, ……., M), and the average image of all training samples is denoted by \( A \). Then, \( G_i \) can be evaluated by

\[ G_i = \frac{1}{M} \sum_{j=1}^{M} (A_j - \overline{A})^T (A_j - \overline{A}) \] (5)

Alternately, the criterion in (2) can be expressed by

\[ J(X) = X^T G_i X \] (6)

Where \( X \) is a unitary column vector. This criterion is called the generalized total scatter criterion. The unitary vector \( X \) that maximizes the criterion is called the optional projection axis. Intuitively, this means that the total scatter of the projected samples is maximized after the projection of an image matrix onto \( X \).

The optimal projection axis \( X_{opt} \) is the unitary vector that maximizes \( J(X) \), i.e., the eigenvector of \( G_i \) corresponding to the largest eigenvalue [17]. In general, it is not enough to have only one optimal projection axis. We usually need to select a set of projection axes, \( X_1, ..., X_d \), subject to the orthonormal constraints and maximizing the criterion \( J(X) \), that is,

\[ \{X_1, ..., X_d\} = \arg \max J(X) \]
\[
X_i^T X_j = 0, i \neq j, i, j = 1, \ldots, d
\] (7)

In fact, the optimal projection axes, \(X_1, \ldots, X_d\), are the orthonormal eigenvectors of \(G_t\) corresponding to the first \(d\) largest eigenvalues.

A] Feature Extraction

The optimal projection vectors of 2DPCA, \(X_1, \ldots, X_d\), are used for feature extraction. For a given image sample \(A\), let
\[
Y_k = A X_k, K = 1,2, \ldots, d
\] (8)

Then, we obtain a family of projected feature vectors, \(Y_1, \ldots, Y_d\), which are called the principal component (vectors) of the sample image \(A\). It should be noted that each principal component of 2DPCA is a vector, whereas the principal component of PCA is a scalar.

The principal component vectors obtained are used to form an \(m \times d\) matrix \(B = [Y_1, \ldots, Y_d]\), which is called the feature matrix or feature image of the image sample \(A\).

B] Classification Method

After a transformation by 2DPCA, a feature matrix is obtained for each image. Then, a nearest neighbor classifier is used for classification. Here, the distance between two arbitrary feature matrices, \(B_i = [Y_{i1}^{(1)}, Y_{i2}^{(1)}, \ldots, Y_{id}^{(1)}]\) and \(B_j = [Y_{j1}^{(1)}, Y_{j2}^{(1)}, \ldots, Y_{jd}^{(1)}]\), is defined by
\[
d(B_i, B_j) = \sum_{k=1}^{d} \| Y_k^{(i)} - Y_k^{(j)} \|_2, \ldots
\] (9)

where \(\| Y_k^{(i)} - Y_k^{(j)} \|_2\) denotes the Euclidean distance between the two principal component vectors \(Y_k^{(i)}\) and \(Y_k^{(j)}\).

Suppose that the training samples are \(B_1, B_2, \ldots, B_M\) (where \(M\) is the total number of training samples), and that each of these samples is assigned a given identity (class) \(w_k\). Given a test sample \(B\), if \(d(B, B_i) = \min d(B, B_j)\) and \(B_i \in w_k\), then the resulting decision is \(B \in w_k\).

III.2DPCA BASED IMAGE RECONSTRUCTION

In the Eigenfaces method, the principal components and eigen vectors (eigenfaces) can be combined to reconstruct the image of a face. Similarly, 2DPCA can be used to reconstruct a face image in the following way.

Suppose the orthonormal eigenvectors corresponding to the first \(d\) largest eigenvectors of the image covariance matrix \(G_t\) are \(X_1, \ldots, X_d\). After the image sample are projected onto these axes, the resulting principal component vectors are \(Y_k = AX_k (k = 1,2, \ldots, d)\). Let \(V = [Y_1, \ldots, Y_d]\) and \(U = [X_1, \ldots, X_d]\), then
\[
V = AU.
\] (10)

Since \(X_1, \ldots, X_d\) are orthonormal, from (10), it is easy to obtain the reconstructed image of sample \(A\):
\[
\overline{A} = VU^T = \sum_{k=1}^{d} Y_k X_k^T
\] (11)

Let \(\overline{A}_k = Y_k X_k^T (k = 1,2, \ldots, d)\), which is of the same size as image \(A\), and represents the reconstructed sub-image of \(A\). That is image \(A\) can be approximately reconstructed by adding up the first \(d\) sub-images. In particular, when the selected number of principal component vectors \(d = n\) (\(n\) is the total number of eigenvectors of \(G_t\)) we have \(\overline{A} = A\), i.e., the image is completely reconstructed by its principal component vectors without any loss of information. Otherwise, if \(d < n\), the reconstructed image \(\overline{A}\) is an approximation for \(A\).

Reconstruction is also done after Noise(different intensities) is added onto an Image.

1) Reading the learning set images of different classes or persons (each image of person is captured in different orientation) and say \(x_{MN}\).
2) Calculate the means (\(u_M (a*b)\)) of images of different classes using the formula given below
\[
u_M = x_{M1}+x_{M2}+\ldots+x_{MN}
\]

where \(x_{MN}\) is \(N\)th image of class \(M\).
3) Find the scattering matrix (\(st_M(a*b)\)) using the formula given below
\[
st_M= \sum_{i=1}^{a} \sum_{j=1}^{b} [(x_{ij}-ui)(x_{ij}-ui)^T].
\]
4) Find the eigen values and eigen vectors of stM. Denote the eigen vector matrix with $vM(a*b)$ and eigen values with $dM(a*1)$.

5) Select first ‘d’ highest eigen values (say d=10)
6) Find the projection matrix $wM(a*d)$
   
   
   
   
   where a = no. of rows, b= no. of columns, b1=b-d;

7) Find the feature matrix $bMN(d*b)$ using
   
where $bMN= wM^T xMN$.

8) Read the test image and say xt(a*b).
9) Find feature matrices of test image $btM(d*b)$ for M classes
   
   
   
   

10) Calculate minimum distances between feature matrices of test image to feature matrix of Nth image of class M and assign the results to the matrix $dist(M,N)$ of a*b

   

11) Find the average minimum distance for individual class using the formula given

   

B

12) Now which class is having less average minimum distance is the class of the test image

IV. EXPERIMENTS AND ANALYSIS

The proposed 2DPCA method was used for face recognition and tested on three well-known face image databases (ORL, AR, and Yale). The ORL database was used to evaluate the performance of 2DPCA under conditions where the pose and sample size are varied. The AR database was employed to test the performance of the system under conditions where there is a variation over time, in facial expressions, and in lighting conditions. The Yale database was used to examine the system performance when both facial expressions and illumination are varied.

V. 2DPCA-BASED IMAGE RECONSTRUCTION

ALGORITHM

In the Eigen method, the principal components and eigen vectors(eigen faces) can be combined to reconstruct the image of a face.Similarly,2DPCA can be used to reconstruct a face image in the following way.

Suppose the orthonormal eigenvectors corresponding to the first d largest eigenvectors of the image covariance matrix Gt are $X_1,........,X_{10}$.After the image samples are projected onto these axes, the resulting principal component vectors are

$Y_k = AX_k$ (k=1,2,........,d).

Let $V=[Y_1,........,Y_d]$ and $U=[X_1,........,X_d]$ then

$V = AU$

Since $X_1,........,X_d$ are orthonormal ,Hence the reconstructed image of a Sample A:

$\bar{A} = VU = \sum Y_k X_k$

Noise is also added in the image and the Image is also Reconstructed with different intensities if Noise.

1. Read all sample images from training set.
   Let the images be $A_1,A_2,........,A_6$.
2. Compute the Average(avg) of the training sample images,

   $\bar{A} = A_1+A_2+A_3+A_4+A_5+A_6/6$

3. Obtain Image co-variance matrix

   $G=((\bar{A}_1 - A)^T (\bar{A}_1 - A) + (\bar{A}_2 - A)^T (\bar{A}_2 - A)
   + (\bar{A}_3 - A)^T (\bar{A}_3 - A) + (\bar{A}_4 - A)^T (\bar{A}_4 - A) + (\bar{A}_5 - A)^T (\bar{A}_5 - A)
   + (\bar{A}_6 - A)^T (\bar{A}_6 - A)) / 6$

4. Compute $X_1,........,X_d$ which are the orthonormal eigen vectors of Gt corresponding to first d largest eigen values.

5. Compute the Reconstructed Image

   

$A = \sum_{k=1}^{d} Y_k X_k$
VI. 2DPCA BASED IMAGE RECOGNITION

ALGORITHM

1. Read all sample images from training set. Let the images be A1,A2……..A6.
2. Compute the Average(avg) of the training sample images,

\[ \bar{A} = \frac{A1+A2+A3+A4+A5+A6}{6} \]
3. Obtain Image co-variance matrix

\[ G = ( (\bar{A}1 - A)^T (\bar{A}1 - A) + (\bar{A}2 - A)^T (\bar{A}2 - A) + (\bar{A}3 - A)^T (\bar{A}3 - A) + (\bar{A}4 - A)^T (\bar{A}4 - A) + (\bar{A}5 - A)^T (\bar{A}5 - A) + (\bar{A}6 - A)^T (\bar{A}6 - A)) / 6 \]
4. Compute X1………….Xd which are the orthonormal eigen vectors of Gt corresponding to first d largest eigen values.

a) Assume 10 largest eigen values, hence d=10 i.e. (X1,……..X10)
5. Compute the Principal Component Vectors

\[ Y_k = AX_k \quad k=1,2,3,…….,10 \]
6. Arrange Y_k the feature matrix B

\[ B = [ Y1,Y2,……..,Y10 ] \]
7. Compute feature matrix B for the images in the database
8. Compute the feature matrix B for the unknown image.
9. Calculate the Euclidian distance between the feature matrix of each Database image with the feature matrix of unknown image.
10. Minimum Distance corresponds to the image recognized.

VII. RESULTS

Sample Database of Faces are taken containing a set of face images taken. The database was used in the context of a face recognition project carried out in collaboration with the Speech, Vision and Robotics Group of the Cambridge University Engineering Department.

There are ten different images of each of 40 distinct subjects. For some subjects, the images were taken at different times, varying the lighting, facial expressions (open / closed eyes, smiling / not smiling) and facial details (glasses / no glasses). All the images were taken against a dark homogeneous background with the subjects in an upright, frontal position (with tolerance for some side movement).

The files are in PGM format, and can conveniently be viewed on UNIX (TM) systems using the 'xv' program. The size of each image is 92x112 pixels, with 256 grey levels per pixel. The images are organised in 40 directories (one for each subject), which have names of the form sX, where X indicates the subject number (between 1 and 40). In each of these directories, there are ten different images of that subject, which have names of the form Y.pgm, where Y is the image number for that subject (between 1 and 10).

Sample of Few Database Images

Fig 1- Few Database Images of Set 1
RECONSTRUCTION for Image shown in Fig 1 using 2DPCA

Fig 4- Original Image & Reconstructed Images
(a) Original Image
(b) Reconstructed Image using 2 Vectors
(c) Reconstructed Image using 5 Vectors
(d) Reconstructed Image using 10 Vectors
(e) Reconstructed Image using 15 Vectors
(f) Reconstructed Image using 20 Vectors
(g) Reconstructed Image using 25 Vectors
(h) Reconstructed Image using 30 Vectors

Table-1 Reconstruction for Figure-1

<table>
<thead>
<tr>
<th>S.No</th>
<th>No. of Principal Component Vectors</th>
<th>Reconstruction Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>2</td>
<td>2.3702e+006</td>
</tr>
<tr>
<td>2.</td>
<td>5</td>
<td>2.3108e+006</td>
</tr>
<tr>
<td>3.</td>
<td>10</td>
<td>2.2013e+006</td>
</tr>
<tr>
<td>4.</td>
<td>15</td>
<td>2.1672e+006</td>
</tr>
<tr>
<td>5.</td>
<td>20</td>
<td>2.0717e+006</td>
</tr>
<tr>
<td>6.</td>
<td>25</td>
<td>2.0717e+006</td>
</tr>
<tr>
<td>7.</td>
<td>30</td>
<td>2.0415e+006</td>
</tr>
</tbody>
</table>

VIII. CONCLUSION

In order to test 2DPCA and evaluate its performance, we performed experiments on image data bases such as ORL database. When a small number of principal components of PCA are used to represent as image the mean square error (MSE) between the reconstructed image and the original image is minimal. We investigated this property for 2DPCA. The result shows that the Reconstruction
image is clear as the numbers of Principal Component Vectors are increased. We also developed methods for reducing the dimensionality of 2DPCA required for image Recognition.

Finally there are still some aspects of 2DPCA that deserve further study. When a small number of principal components of PCA are used to represent an image, the mean square error (MSE) between the approximation and the original image is minimal in addition it needs more coefficients for image representation.

REFERENCES


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